Semiclassical vortex confinement picture of 4d YM theory on $\mathbb{R}^2 \times T^2$ and its relation to anomaly matching

Yuya Tanizaki (Yukawa institute, Kyoto)

with Mithat Ünsal (NCSU)

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Adiabatic Continuity conjecture (YT, Ünsal)

YM, QCD on $\mathbb{R}^2 \times T^2$ w/ 't Hooft flux $\Longleftrightarrow$ YM, QCD on $\mathbb{R}^4$

Adiabatic Continuity

reliable semiclassics with center vortices

strong-couplings

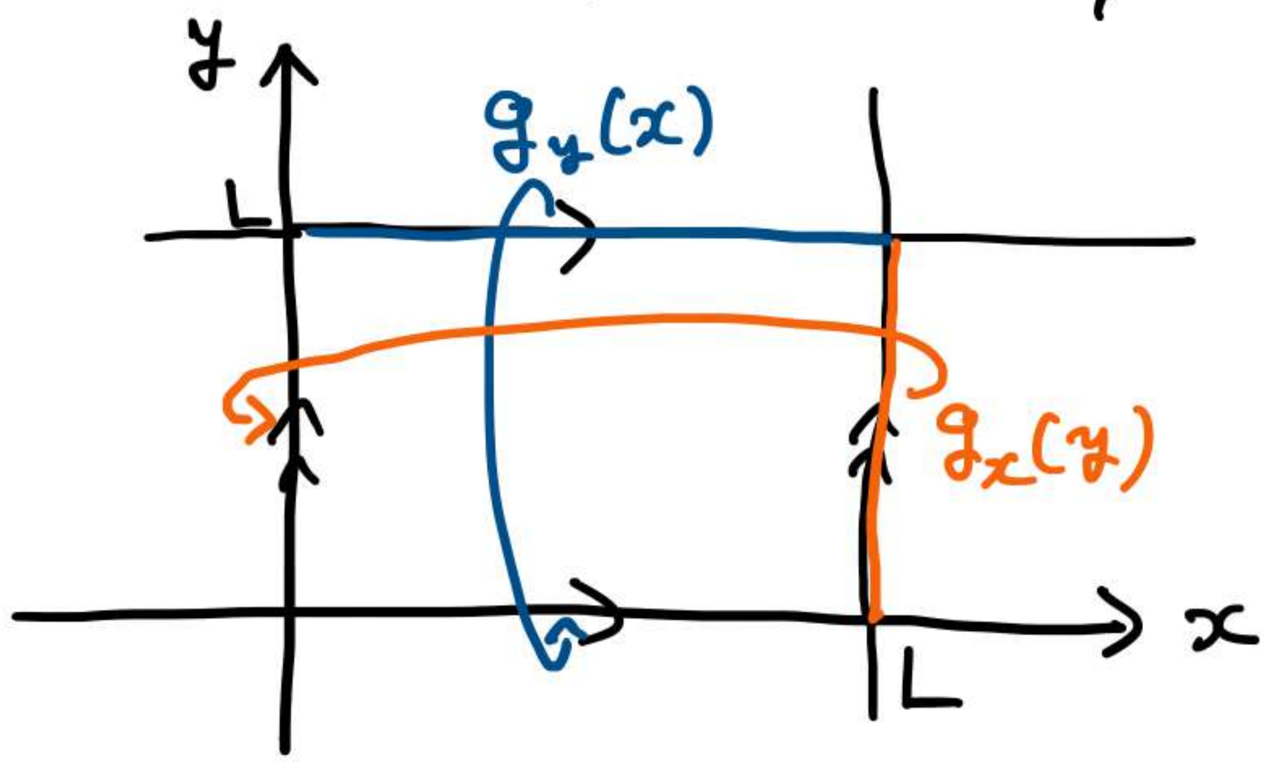
$\Downarrow$

- $\boxed{\text{(YM theory)}}$ $\quad E_k(\theta) \sim -\Lambda^2 (\Lambda L)^{\frac{5}{3}} \cos\left(\frac{\theta - 2\pi k}{N}\right)$ (Multi-branch confining vacua)
- ($\mathcal{N}=1$ SYM) $\quad \langle \mathrm{tr}(\lambda\lambda) \rangle_k \sim \Lambda^3 e^{i \cdot \frac{\theta - 2\pi k}{N}}$
- (QCD w/ non-commuting flavor twist $(N_c = N_f = N)$) $\quad \langle \mathrm{tr}_{cf}(\bar{\psi}_L)\, \mathrm{tr}_{cf}(\psi_R) \rangle \sim \Lambda^3 e^{i \frac{\theta - 2\pi k}{N}}$

(Spontaneous breaking of discrete chiral symmetry)

- (QCD w/ $U(1)_B$ monopole flux) $\quad S_{\mathrm{eff}} \sim \int \left[\underbrace{|dU|^2 + \frac{1}{12\pi} \mathrm{tr}(U^\dagger dU)^3}_{U(N_f),\ \mathrm{WZW}} + \underbrace{\chi_{top} (i \ln \det U - \theta)^2}_{\eta' \text{ mass consistent with Witten-Veneziano formula}} \right]$

't Hooft twisted boundary condition on $T^2 \ni (x,y)$



$$x \sim x+L$$
$$y \sim y+L$$

$\phi(x,y)$: adjoint matter field $\quad (\mathrm{Ad}(U^\dagger)\phi = U^\dagger \phi U)$

$$\phi(L,y) = \mathrm{Ad}(g_x^\dagger(y))\,\phi(0,y)$$

$$\phi(x,L) = \mathrm{Ad}(g_y^\dagger(x))\,\phi(x,0)$$

Uniqueness of the matter wavefunction requires

$$g_x^\dagger(L)\, g_y^\dagger(0) = g_y^\dagger(L)\, g_x^\dagger(0)\, e^{\frac{2\pi i}{N} n_{xy}} .$$

$\uparrow$ 't Hooft flux.

(cf When fundamental matters exist, the condition becomes
$$g_x^\dagger(L)\, g_y^\dagger(0) = g_y^\dagger(L)\, g_x^\dagger(0) .$$
)

Perturbative analysis of SU(N) YM on $\mathbb{R}^2 \times T^2$ w/ 't Hooft flux.

- $\mathbb{Z}_N \times \mathbb{Z}_N$ center symmetry is unbroken.

- 2d gluons are gapped.

$\Leftarrow$ Polyakov loops along T^2 are adjoint Higgs fields for $\mathbb{R}^2$.

$$P_3 = S, \quad P_4 = C \quad \text{gives}$$

$$SU(N) \xrightarrow{\text{Higgsing}} \mathbb{Z}_N.$$

Weak-coupling analysis is free from IR divergences.

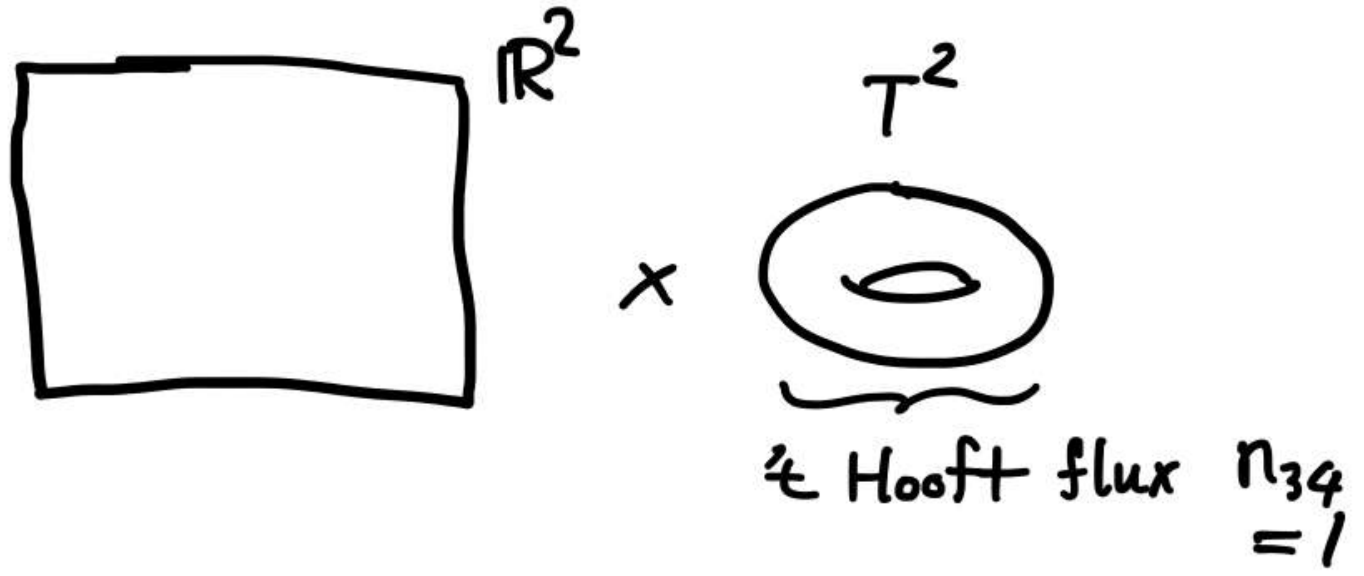
- However, <u>Wilson loops inside $\mathbb{R}^2$ obey perimeter laws</u>.

$\Downarrow$

We have to resolve this problem to achieve adiabatic continuity. $\Longrightarrow$ Center vortex

Center vortex as a fractional instanton on $\mathbb{R}^2 \times T^2$

In this setup, the minimal topological charge is given by



$$\boxed{Q_{top} = \frac{1}{8\pi^2}\int \mathrm{tr}(F \wedge F) = \frac{1}{N}}$$

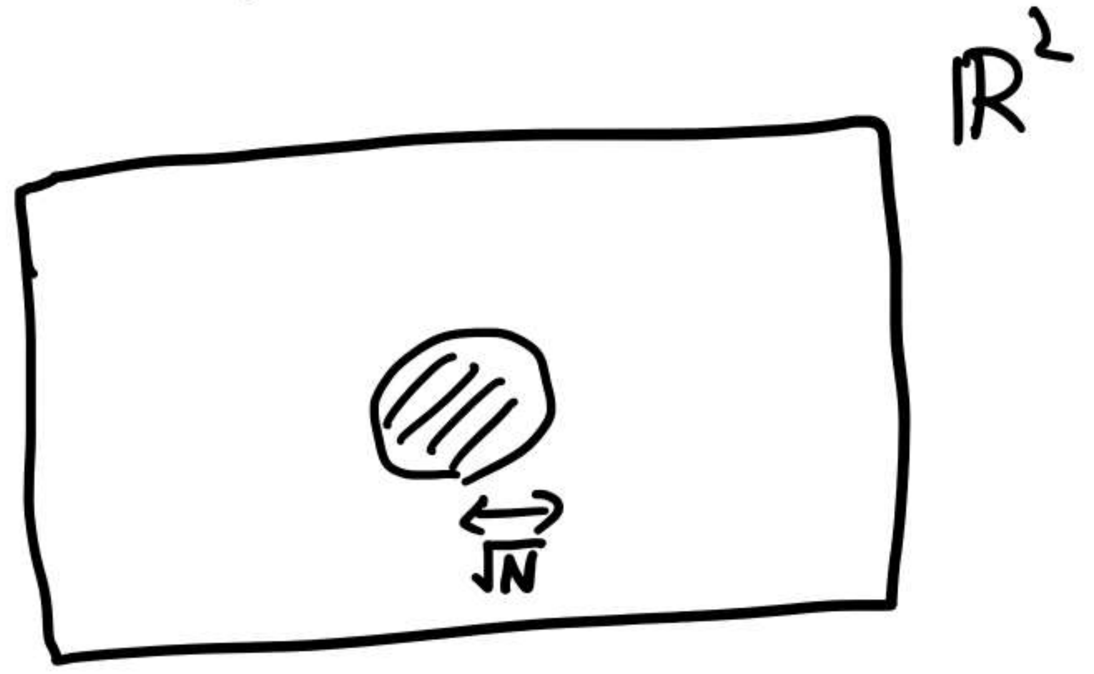
(More precisely, $Q_{top} \in \frac{1}{N}\left(\frac{-\epsilon_{\mu\nu\rho\sigma} n_{\mu\nu} n_{\rho\sigma}}{8}\right) + \mathbb{Z}$ (van Baal '82))

If there exists a self-dual configuration, its Yang-Mills action becomes

$$\boxed{S_{YM} = \frac{8\pi^2}{g^2} |Q_{top}| = \frac{8\pi^2}{g^2 \cdot N}}$$

Gonzalez-Arroyo, Montero '98, Montero '99 numerically confirmed such a classical solution exists:

center vortex
or fractional instanton.



$$\left(\begin{array}{l} Q_{top} = \frac{1}{N} \\ S_{YM} = \frac{8\pi^2}{g^2 N} \end{array}\right)$$

(cf. Garcia Perez, Gonzalez-Arroyo, '92, Itou '18)

Partition funcion on $\underbrace{M_2}_{\to \mathbb{R}^2} \times T^2$ & θ-dependence

To make the computation well-defined, we compactify $\mathbb{R}^2$ to some closed 2-manifold M_2.

Using the 1-loop vertex of the center vortex

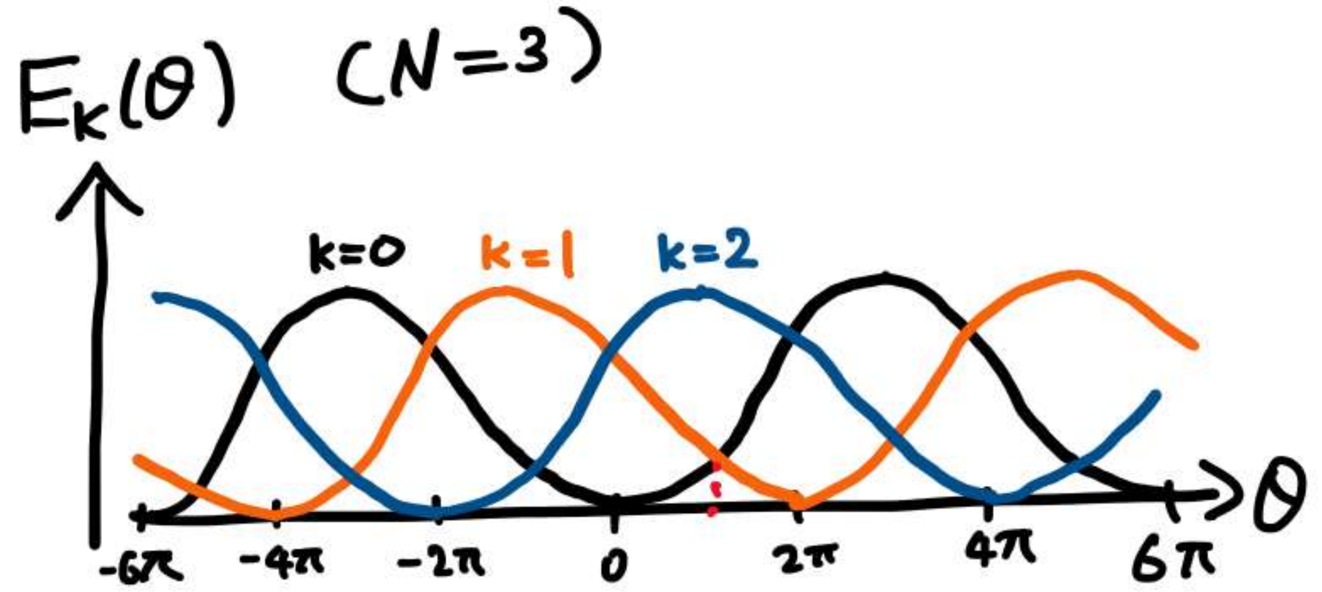
$$K \cdot e^{-\frac{8\pi^2}{g^2 N} + i\frac{\theta}{N}},$$

we have

$$Z(\theta) = \sum_{n, \bar{n} \geq 0} \frac{\delta_{n-\bar{n} \in N\mathbb{Z}}}{n!\, \bar{n}!} \Big(\underbrace{V \cdot K e^{-\frac{8\pi^2}{g^2 N} + i\frac{\theta}{N}}}_{\text{vortex}} \Big)^{n} \Big(\underbrace{V \cdot K e^{-\frac{8\pi^2}{g^2 N} - i\frac{\theta}{N}}}_{\text{anti-vortex}} \Big)^{\bar{n}}$$

$$= \sum_{k=0}^{N-1} \exp\Big[-V \Big(\underbrace{-2K e^{-\frac{8\pi^2}{g^2 N}} \cos\Big(\frac{\theta - 2\pi k}{N}\Big)}_{E_k(\theta)\,:\ \text{Ground-state energy densities}} \Big) \Big]$$

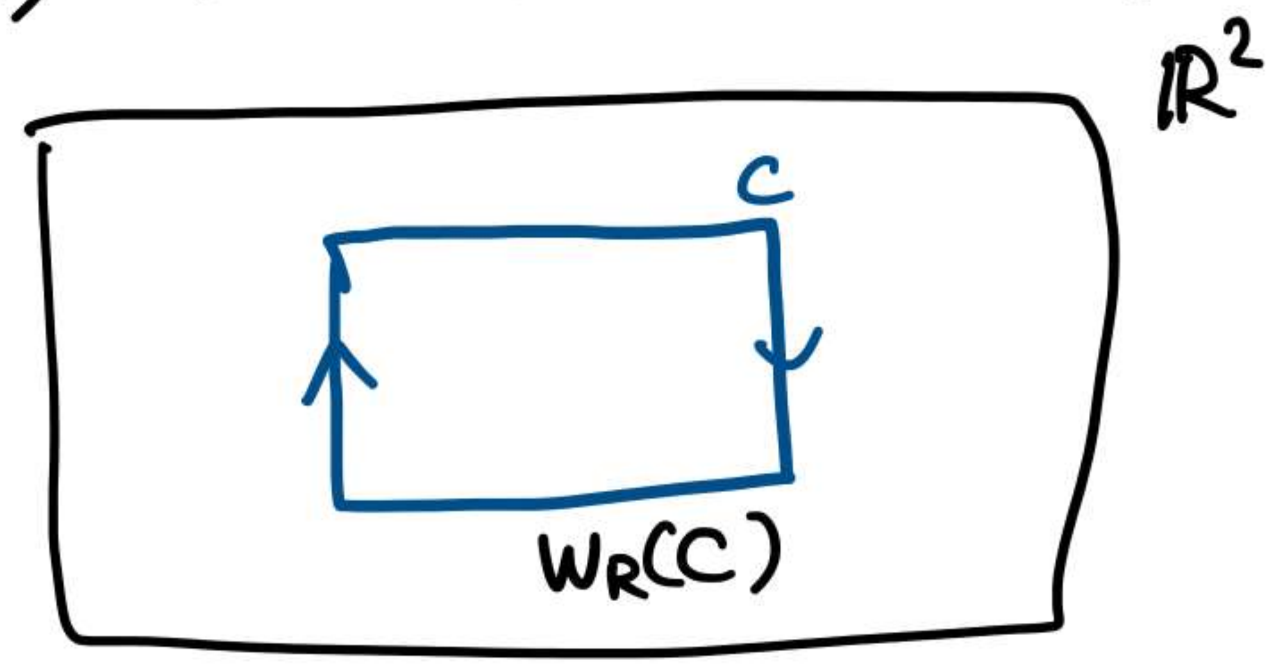
$\Rightarrow$
- N-branch structure of ground states.
- Each branch has a fractional θ-dependence.



Confinement of Wilson loops inside $\mathbb{R}^2$

Using the dilute gas approximation, we can also compute

$$\langle W_R(C) \rangle =$$



$$= \exp\left(- \text{Area} \times T_R(\theta)\right).$$

For $-\pi < \theta < \pi$, the string tension is given by

$$T_R(\theta) = E_{|R|}(\theta) - E_0(\theta),$$

where $|R|$ is the N-ality of the representation.

In particular, at $\theta = 0$,

$$T_R(\theta=0) \sim \Lambda^2 (\Lambda L)^{\frac{5}{3}} \sin^2\left(\frac{\pi |R|}{N}\right).$$

Anomaly matching

4d SU(N) YM theory has an 't Hooft anomaly: (Gaiotto, Kapustin, Komargodski, Seiberg '17)

$$Z_{\theta+2\pi}[B] = e^{i\frac{N}{4\pi}\int B\wedge B} Z_\theta[B]$$

$\Longrightarrow$ Confinement implies the multi-branch structure of vacua.

With T^2-compactification, $(\mathbb{Z}_N^{[1]})_{4d}$ splits into $(\mathbb{Z}_N^{[1]})_{2d}\times \mathbb{Z}_N^{[0]}\times \mathbb{Z}_N^{[0]}$.

When 't Hooft flux n_{34} (mod N) is introduced, the 4d anomaly becomes

$$Z_{\theta+2\pi}[B_{2d}, A, A'] = e^{i\left(n_{34}\int B - \frac{N}{2\pi}\int A\wedge A'\right)} Z_\theta[B, A, A'].$$

$\Longrightarrow$ When $\gcd(n_{34}, N)=1$ (especially when $n_{34}=1$),

confinement implies the multi-branch structure. (cf. YT, Misumi, Sakai, '17. Yamazaki '17.)

Both properties are obtained by the center vortex.

SUMMARY

$$\underbrace{\text{YM, QCD on } \mathbb{R}^2 \times T^2 \text{ w/ 't Hooft flux}}_{\text{Semiclassics w/ center vortices}} \quad \underset{\text{Adiabatic Continuity}}{\Longleftrightarrow} \quad \underbrace{\text{YM, QCD on } \mathbb{R}^4}_{\text{Strong-couplings}}$$

- $\boxed{\text{(YM theory)}}$ $\quad E_k(\theta) \sim -\Lambda^2 (\Lambda L)^{\frac{5}{3}} \cos\left(\frac{\theta - 2\pi k}{N}\right)$ (Multi-branch vacua)
- ($\mathcal{N}=1$ SYM) $\quad \langle \mathrm{tr}(\lambda\lambda) \rangle \sim \Lambda^3 e^{i\frac{\theta-2\pi k}{N}}$
- $\begin{pmatrix}\text{QCD w/ non-commuting} \\ \text{flavor twist } (N_c = N_f = N)\end{pmatrix}$ $\quad \underline{\langle \mathrm{tr}_{cf}(\bar\Psi)\,\mathrm{tr}_{cf}(\Psi) \rangle} \sim \underline{\underline{\Lambda^3 e^{i\frac{\theta-2\pi k}{N}}}}$

(Discrete chiral SSB)

- $\boxed{\begin{pmatrix}\text{QCD w/} \\ U(1)_B \text{ monopole flux}\end{pmatrix}}$ $\quad S_{\mathrm{eff}} \sim \int \left(|dU|^2 + \frac{1}{12\pi}\mathrm{tr}((U^\dagger dU)^3) + \chi_{top}\,(i \ln\det U - \theta)^2 \right)$

η' mass from Witten-Veneziano formula